4. Measurement of Angles

Exercise 4.1

1. Question

Find the degree measure corresponding to the following radian measures (Use $\pi = \frac{22}{7}$)

(i) $\frac{9\pi}{5}$ (ii) $-\frac{5\pi}{6}$ (iii) $\left(\frac{18\pi}{5}\right)^c$ (iv) $(-3)^c$ (v) 11^c (vi) 1^c

Answer

We know that π rad = 180° ⇒ 1 rad = 180°/ π

(i) Given $\frac{9\pi}{5}$

$= \left(\frac{180}{\pi} \times \frac{9\pi}{5}\right)^\circ$

= (36 × 9) °

= 324°

(ii) Given $-\frac{5\pi}{6}$

$= \left(\frac{180}{\pi} \times -\frac{5\pi}{6}\right)^\circ$

= (30 × -5) °

= -(150) °

(iii) Given $\left(\frac{18\pi}{5}\right)^c$

$= \left(\frac{180}{\pi} \times \frac{18\pi}{5}\right)^\circ$

= (36 × 18) °

= 648°

(iv) Given $(-3)^c$

$= \left(\frac{180}{\pi} \times -3\right)^\circ$

$= \left(\frac{180}{22} \times 7 \times -3\right)^\circ$

$= \left(-\frac{3780}{22}\right)^\circ$

$= \left(-171\frac{18}{22}\right)^\circ$

$= \left(-171^\circ \left(\frac{18}{22} \times 60\right)'\right)$

$= \left(-171^\circ \left(49\frac{1}{11}\right)'\right)$

$$= \left(-171°49'\left(\frac{1}{11} \times 60\right)'\right)$$

= -(171° 49’ 5.45”)

≈ -(171° 49’ 5”)

(v) Given 11^c

$$= \left(\frac{180}{\pi} \times 11\right)°$$

$$= \left(\frac{180}{22} \times 7 \times 11\right)°$$

= (90 × 7) °

= 630°

(vi) Given 1^c

$$= \left(\frac{180}{\pi} \times 1\right)°$$

$$= \left(\frac{180}{22} \times 7 \times 1\right)°$$

$$= \left(\frac{1260}{22}\right)°$$

$$= \left(57\frac{3}{11}\right)°$$

$$= \left(57°\left(\frac{3}{11} \times 60\right)'\right)$$

$$= \left(57°\left(16\frac{4}{11}\right)'\right)$$

$$= \left(57°16'\left(\frac{4}{11} \times 60\right)'\right)$$

= (57° 16’ 21.81”)

≈ (57° 16’ 22”)

2. Question

Find the radian measure corresponding to the following degree measures:

(i) 300^0 (ii) 35^0 (iii) -56^0 (iv)135^0 (v) -300^0

(vi) 7^0 30’ (vii) 125^0 30’ (viii) -47^0 30’

Answer

We know that 180° = π rad ⇒ 1° = π/ 180 rad

(i) Given 300°

$$= \left(300 \times \frac{\pi}{180}\right) \text{rad}$$

$$= \frac{5\pi}{3} \text{rad}$$

(ii) Given 35°

$= \left(35 \times \frac{\pi}{180}\right)$ rad

$= \frac{7\pi}{36}$ rad

(iii) Given -56°

$= \left(-56 \times \frac{\pi}{180}\right)$ rad

$= -\frac{14\pi}{45}$ rad

(iv) Given 135°

$= \left(135 \times \frac{\pi}{180}\right)$ rad

$= \frac{3\pi}{4}$ rad

(v) Given -300°

$= \left(-300 \times \frac{\pi}{180}\right)$ rad

$= -\frac{5\pi}{3}$ rad

(vi) Given 7° 30'

We know that 30' = (1/2)°

∴ 7° 30' = (7 1/2) °

$= \left(\frac{15}{2}\right)^\circ$

$= \left(\frac{15}{2} \times \frac{\pi}{180}\right)$ rad

$= \left(\frac{\pi}{24}\right)$ rad

(vii) Given 125° 30'

We know that 30' = (1/2)°

∴ 125° 30' = (1251/2) °

$= \left(\frac{251}{2}\right)^\circ$

$= \left(\frac{251}{2} \times \frac{\pi}{180}\right)$ rad

$= \left(\frac{251\pi}{360}\right)$ rad

(viii) Given -47° 30'

We know that 30' = (1/2)°

∴ -47° 30' = -(471/2) °

$= -\left(\frac{95}{2}\right)^\circ$

$= -\left(\frac{95}{2} \times \frac{\pi}{180}\right)$ rad

$= -\left(\frac{19\pi}{72}\right)$ rad

3. Question

The difference between the two acute angles of a right-angled triangle is 2π/5 radians. Express the angles in degrees.

Answer

Given the difference between the two acute angles of a right-angled triangle is 2π/5 radians.

We know that π rad = 180° ⇒ 1 rad = 180°/ π

Given $\frac{2\pi}{5}$

$= \left(\frac{180}{\pi} \times \frac{2\pi}{5}\right)^{\circ}$

= (36 × 2) °

= 72°

Let one acute angle be x° and the other acute angle be 90° - x°.

Then,

⇒ x° - (90° - x°) = 72°

⇒ 2x° - 90° = 72°

⇒ 2x° = 72° + 90°

⇒ 2x° = 162°

⇒ x° = 162°/ 2

∴ x° = 81° and 90° - x° = 90° - 81° = 9°

4. Question

One angle of a triangle is $\frac{2}{3}$x grades, and another is $\frac{3}{2}$x degrees while the third is $\frac{\pi x}{75}$ radians. Express all the angles in degrees.

Answer

Given one angle of a triangle is 2x/3 grades and another is 3x/2 degree while the third is πx/75 radians.

We know that $1 \text{ grad} = \left(\frac{9}{10}\right)^{\circ}$

$\Rightarrow \frac{2}{3}x \text{ grad} = \left(\frac{9}{10}\right)\left(\frac{2}{3}x\right)^{\circ} = \frac{3}{5}x^{\circ}$

We know that π rad = 180° ⇒ 1 rad = 180°/ π

Given $\frac{\pi x}{75}$

$= \left(\frac{180}{\pi} \times \frac{\pi x}{75}\right)^{\circ}$

$= \left(\frac{12}{5}x\right)^{\circ}$

We know that the sum of the angles of a triangle is 180°.

$$\Rightarrow \frac{3}{5}x^\circ + \frac{3}{2}x^\circ + \frac{12}{5}x^\circ = 180^\circ$$

$$\Rightarrow \frac{6+15+24}{10}x^\circ = 180^\circ$$

⇒ 45 x° = 180° × 10°

⇒ 45 x° = 1800°

⇒ x° = 1800°/ 45°

∴ x = 40°

∴ The angles of the triangle are

$$\Rightarrow \frac{3}{5}x^\circ = \frac{3}{5} \times 40^\circ = 24^\circ$$

$$\Rightarrow \frac{3}{2}x^\circ = \frac{3}{2} \times 40^\circ = 60^\circ$$

$$\Rightarrow \frac{12}{5}x^\circ = \frac{12}{5} \times 40^\circ = 96^\circ$$

5. Question

Find the magnitude, in radians and degrees, of the interior angle of a regular:

(i) Pentagon (ii) Octagon (iii) Heptagon (iv) Duodecagon.

Answer

We know that the sum of the interior angles of a polygon = (n – 2) π

And each angle of polygon $= \frac{\text{sum of interior angles of polygon}}{\text{number of sides}}$

(i) Pentagon

Number of sides in pentagon = 5

Sum of interior angles of pentagon = (5 – 2) π = 3π

∴ Each angle of pentagon $= \frac{3\pi}{5} \times \frac{180^\circ}{\pi} = 108^\circ$

(ii) Octagon

Number of sides in octagon = 8

Sum of interior angles of octagon = (8 – 2) π = 6π

∴ Each angle of octagon $= \frac{6\pi}{8} \times \frac{180^\circ}{\pi} = 135^\circ$

(iii) Heptagon

Number of sides in heptagon = 7

Sum of interior angles of heptagon = (7 – 2) π = 5π

∴ Each angle of heptagon $= \frac{5\pi}{7} \times \frac{180^\circ}{\pi} = \frac{900}{7}^\circ = 128^\circ 34' 17''$

(iv) Duodecagon

Number of sides in duodecagon = 12

Sum of interior angles of duodecagon = (12 – 2) π = 10π

∴ Each angle of duodecagon $= \frac{10\pi}{12} \times \frac{180^\circ}{\pi} = 150^\circ$

6. Question

The angles of a quadrilateral are in A.P., and the greatest angle is 120^0. Express the angles in radians.

Answer

Let the angles of quadrilateral be (a – 3d) °, (a – d) °, (a + d) ° and (a + 3d) °.

We know that the sum of angles of a quadrilateral is 360°.

⇒ a – 3d + a – d + a + d + a + 3d = 360°

⇒ 4a = 360°

∴ a = 90°

Given the greatest angle = 120°

⇒ a + 3d = 120°

⇒ 90° + 3d = 120°

⇒ 3d = 120° - 90°

⇒ 3d = 30°

⇒ d = 10°

Hence, the angles are:

⇒ (a – 3d) ° = 90° - 30° = 60°

⇒ (a – d) ° = 90° - 10° = 80°

⇒ (a + d) ° = 90° + 10° = 100°

⇒ (a + 3d) ° = 120°

Angles of quadrilateral in radians:

$$\Rightarrow \left(60 \times \frac{\pi}{180}\right) \text{ rad} = \frac{\pi}{3}\text{rad}$$

$$\Rightarrow \left(80 \times \frac{\pi}{180}\right) \text{ rad} = \frac{4\pi}{9}\text{rad}$$

$$\Rightarrow \left(100 \times \frac{\pi}{180}\right) \text{ rad} = \frac{5\pi}{9}\text{rad}$$

$$\Rightarrow \left(120 \times \frac{\pi}{180}\right) \text{ rad} = \frac{2\pi}{3}\text{rad}$$

7. Question

The angles of a triangle are in A.P., and the number of degrees in the least angle is to the number of degrees in the mean angle as 1:120. Find the angle in radians.

Answer

Let the angles of the triangle be (a – d) °, a° and (a + d) °.

We know that the sum of the angles of a triangle is 180°.

⇒ a – d + a + a + d = 180°

⇒ 3a = 180°

∴ a = 60°

Given $\frac{\text{Number of degrees in the least angle}}{\text{Number of degrees in the mean angle}} = \frac{1}{120}$

$\Rightarrow \frac{a-d}{a} = \frac{1}{120}$

$\Rightarrow \frac{60-d}{60} = \frac{1}{120}$

$\Rightarrow \frac{60-d}{1} = \frac{1}{2}$

⇒ 120 – 2d = 1

⇒ 2d = 119

∴ d = 59.5

Hence, angles are:

⇒ (a – d) ° = 60° – 59.5° = 0.5°

⇒ a° = 60°

⇒ (a + d) ° = 60° + 59.5° = 119.5°

∴ Angles of triangle in radians:

$\Rightarrow \left(0.5 \times \frac{\pi}{180}\right) \text{ rad} = \frac{\pi}{360} \text{rad}$

$\Rightarrow \left(60 \times \frac{\pi}{180}\right) \text{ rad} = \frac{\pi}{3} \text{rad}$

$\Rightarrow \left(119.5 \times \frac{\pi}{180}\right) \text{ rad} = \frac{239\pi}{360} \text{rad}$

8. Question

The angle in one regular polygon is to that in another as 3:2 and the number of sides in first is twice that in the second. Determine the number of sides of two polygons.

Answer

Let the number of sides in the first polygon be 2x and the number of sides in the second polygon be x.

We know that angle of an n-sided regular polygon $= \left(\frac{n-2}{n}\right) \pi \text{ radian}$

⇒ The angle of the first polygon $= \left(\frac{2x-2}{2x}\right) \pi = \left(\frac{x-1}{x}\right) \pi \text{ radian}$

⇒ The angle of the second polygon $= \left(\frac{x-2}{x}\right) \pi \text{ radian}$

Thus,

$\Rightarrow \frac{\left(\frac{x-1}{x}\right)\pi}{\left(\frac{x-2}{x}\right)\pi} = \frac{3}{2}$

$\Rightarrow \frac{x-1}{x-2} = \frac{3}{2}$

⇒ 2x – 2 = 3x – 6

∴ x = 4

Thus,

Number of sides in the first polygon = 2x = 8

Number of sides in the second polygon = x = 4

9. Question

The angles of a triangle are in A.P. such that the greatest is 5 times the least. Find the angles in radians.

Answer

Let the angles of the triangle be (a – d) °, a° and (a + d) °.

We know that the sum of angles of triangle is 180°.

⇒ a – d + a + a + d = 180°

⇒ 3a = 180°

∴ a = 60°

Given greatest angle = 5 × least angle

$$\frac{\text{Greatest angle}}{\text{Least angle}} = 5$$

$$\Rightarrow \frac{a+d}{a-d} = 5$$

$$\Rightarrow \frac{60+d}{60-d} = 5$$

⇒ 60 + d = 300 – 5d

⇒ 6d = 240

∴ d = 40

Hence, angles are:

⇒ (a – d) ° = 60° – 40° = 20°

⇒ a° = 60°

⇒ (a + d) ° = 60° + 40° = 100°

∴ Angles of triangle in radians:

$$\Rightarrow \left(20 \times \frac{\pi}{180}\right) \text{ rad} = \frac{\pi}{9}\text{rad}$$

$$\Rightarrow \left(60 \times \frac{\pi}{180}\right) \text{ rad} = \frac{\pi}{3}\text{rad}$$

$$\Rightarrow \left(100 \times \frac{\pi}{180}\right) \text{ rad} = \frac{5\pi}{9}\text{rad}$$

10. Question

The number of sides of two regular polygons is 5:4 and the difference between their angle is 9^0. Find the number of sides of the polygons.

Answer

Let the number of sides in the first polygon be 5x and the number of sides in the second polygon be 4x.

We know that angle of an n-sided regular polygon $= \left(\frac{n-2}{n}\right) \pi$ radian

⇒ The angle of the first polygon $= \left(\frac{5x-2}{5x}\right) 180°$

⇒ The angle of the second polygon $= \left(\frac{4x-1}{4x}\right) 180°$

Thus,

$$\Rightarrow \left(\frac{5x-2}{5x}\right)180° - \left(\frac{4x-1}{4x}\right)180° = 9$$

$$\Rightarrow 180°\left(\frac{4(5x-2)-5(4x-2)}{20x}\right) = 9$$

$$\Rightarrow \frac{20x-8-20x+10}{20x} = \frac{9}{180}$$

$$\Rightarrow \frac{2}{20x} = \frac{1}{20}$$

$\Rightarrow \frac{2}{x} = 1$

$\therefore x = 2$

Thus,

Number of sides in the first polygon = 5x = 10

Number of sides in the second polygon = 4x = 8

11. Question

A railroad curve is to be laid out on a circle. What radius should be used if the track is to change direction by 25^0 at a distance of 40 meters?

Answer

Given length of arc = 40 m

And θ = 25°

We know that 180° = π rad ⇒ 1° = π/ 180 rad

$$\Rightarrow 25° = \left(25 \times \frac{\pi}{180}\right) = \frac{5\pi}{36}\text{radian}$$

We know that $\theta = \frac{\text{arc}}{\text{radius}}$

$$\Rightarrow \frac{5\pi}{36} = \frac{40}{\text{radius}}$$

$$\Rightarrow \text{Radius} = \frac{40}{\frac{5\pi}{36}}$$

$$= \frac{40 \times 36 \times 7}{5 \times 22}$$

= 91.64 m

So, the radius of the track should be 91.64 m.

12. Question

Find the length which at a distance of 5280 m will subtend an angle of 1’ at the eye.

Answer

Given radius = 5280 m

We know that $\theta = 1' = \left(\frac{1}{60}\right)^{\circ} = \left(\frac{1}{60} \times \frac{\pi}{180}\right)\text{rad}$

And know that $\theta = \frac{\text{arc}}{\text{radius}}$

$$\Rightarrow \frac{1}{60} \times \frac{\pi}{180} = \frac{\text{arc}}{5280}$$

$\therefore \text{arc} = \frac{5280 \times 22}{60 \times 180 \times 7}$

= 1.5365 m

13. Question

A wheel makes 360 revolutions per minute. Through how many radians does it turn in 1 second?

Answer

Given the number of revolutions taken by the wheel in 1 minute = 360

Number of revolution taken by the wheel in 1 second = 360/ 6 = 6

<u>We know that 1 revolution = 2π radians</u>

∴ Number of radians the wheel will turn in 1 second = 6 × 2π = 12π

14. Question

Find the angle in radians through which a pendulum swings if its length is 75 cm and the tip describes an arc of length (i) 10 cm (ii) 15 cm (iii) 21 cm.

Answer

Given radius = 75 cm

We know that $\theta = \frac{\text{arc}}{\text{radius}}$

(i) Given length of arc = 10 cm

$\Rightarrow \theta = \frac{10}{75} = \frac{2}{15}\text{radian}$

(ii) Given length of arc = 15 cm

$\Rightarrow \theta = \frac{15}{75} = \frac{1}{5}\text{radian}$

(iii) Given length of arc = 21 cm

$\Rightarrow \theta = \frac{21}{75} = \frac{7}{25}\text{radian}$

15. Question

The radius of a circle is 30 cm. Find the length of an arc of this circle, if the length of the chord of the arc is 30 cm.

Answer

Let AB be chord and O be the centre if the circle.

Here AO = BO = AB = 30 cm

∴ ΔAOB is an equilateral triangle.

Given radius = 30 cm

And θ = 60°

<u>We know that 180° = π rad</u> ⇒ <u>1° = π/ 180 rad</u>

$\Rightarrow 60° = \left(60 \times \frac{\pi}{180}\right) = \frac{\pi}{3}\text{radian}$

We know that $\theta = \frac{\text{arc}}{\text{radius}}$

$\Rightarrow \frac{\pi}{3} = \frac{\text{arc}}{30}$

$\therefore \text{arc} = \frac{30\pi}{3}$

= 10π cm

16. Question

A railway train is traveling on a circular curve of 1500 meters radius at the rate of 66 km/hr. Through what angle has it turned in 10 seconds?

Answer

Given time is 10 seconds.

And speed = 66 km/h $= \frac{66 \times 1000}{3600} \text{m/s}$

We know that $\text{speed} = \frac{\text{distance}}{\text{time}}$

$\Rightarrow \frac{66 \times 1000}{3600} = \frac{\text{distance}}{\text{time}}$

$\Rightarrow \text{Distance} = \frac{66 \times 1000}{3600} \times 10 = \frac{1100}{6} \text{m}$

Now radius of curve = 1500 m

We know that $\theta = \frac{\text{arc}}{\text{radius}}$

$\Rightarrow \theta = \frac{\frac{1100}{6}}{1500}$

$= \frac{1100}{1500 \times 6} = \frac{11}{90} \text{radian}$

So, the train will turn 11/ 90 radian in 10 seconds.

17. Question

Find the distance from the eye at which a coin of 2 cm diameter should be held so as to conceal the full moon whose angular diameter is 31'.

Answer

Let PQ be the diameter of the coin and E be the eye of the observer.

Let the coin be kept at a distance r from the eye of the observer to hide the moon completely.

$\Rightarrow \theta = 1' = \left(\frac{1}{60}\right)^\circ = \left(\frac{1}{60} \times \frac{\pi}{180}\right) \text{rad}$

We know that $\theta = \frac{\text{arc}}{\text{radius}}$

$\Rightarrow \frac{31}{60} \times \frac{\pi}{180} = \frac{2}{\text{radius}}$

$\therefore \text{radius} = \frac{180 \times 60 \times 2 \times 7}{31 \times 22}$

= 221.7 cm

18. Question

Find the diameter of the Sun in km supposing that it subtends an angle of 32' at the eye of an observer. Given that the distance of the Sun is 91 x 10^6 km.

Answer

Let PQ be the diameter of the sun and E be the eye of the observer.

The distance between the Sun and Earth is large, so we will take PQ as arc PQ.

Given radius = 91×10^6 km

$$\Rightarrow \theta = 32' = \left(\frac{32}{60}\right)^\circ = \left(\frac{32}{60} \times \frac{\pi}{180}\right) \text{rad}$$

We know that $\theta = \frac{\text{arc}}{\text{radius}}$

$$\Rightarrow \frac{32}{60} \times \frac{\pi}{180} = \frac{d}{91 \times 10^6}$$

$$\therefore d = \frac{32 \times 91 \times 10^6 \times 22}{60 \times 180 \times 7}$$

= 847407.4 km

19. Question

If the arcs of the same length in two circles subtend angles 65^0 and 110^0 at the center, find the ratio of their radii.

Answer

Let the angles subtended at the centers by the arcs and radii of first and second circles be θ_1 and r_1 and θ_2 and r_2.

Then,

<u>We know that 180° = π rad</u> ⇒ <u>1° = π/ 180 rad</u>

$$\Rightarrow \theta_1 = 65^\circ = \left(65 \times \frac{\pi}{180}\right) \text{radian}$$

$$\Rightarrow \theta_2 = 110^\circ = \left(110 \times \frac{\pi}{180}\right) \text{radian}$$

We know that $\theta = \frac{\text{arc}}{\text{radius}}$

$$\Rightarrow \theta_1 = \frac{l}{r_1} \text{and } \theta_2 = \frac{l}{r^2}$$

$$\Rightarrow r_1 = \frac{l}{\left(65 \times \frac{\pi}{180}\right)} \text{ and } r_2 = \frac{l}{\left(110 \times \frac{\pi}{180}\right)}$$

$$\therefore \frac{r_1}{r_2} = \frac{\frac{l}{\left(65 \times \frac{\pi}{180}\right)}}{\frac{l}{\left(110 \times \frac{\pi}{180}\right)}}$$

$$= \frac{110}{65}$$

$$= \frac{22}{13}$$

$$\therefore r_1 : r_2 = 22 : 13$$

20. Question

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use $\pi = \frac{22}{7}$).

Answer

Given length of arc = 22 cm

And radius = 100 cm

We know that $\theta = \frac{\text{arc}}{\text{radius}}$

$\Rightarrow \theta = \frac{22}{100}$

$= \frac{11}{50}\text{radian}$

∴ The angle subtended at the centre by the arc:

$\Rightarrow \left(\frac{32}{60} \times \frac{\pi}{180}\right)^{\circ} = \left(\frac{11}{5} \times \frac{18}{220} \times 7\right)^{\circ}$

$= \left(\frac{63}{5}\right)^{\circ}$

= 12° 36'.

MCQ

1. Question

Mark the correct alternative in the following:

If D, G and R denote respectively the number of degrees, grades and radians in an angle, then

A. $\frac{D}{100} = \frac{G}{90} = \frac{2R}{\pi}$

B. $\frac{D}{90} = \frac{G}{100} = \frac{R}{\pi}$

C. $\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$

D. $\frac{D}{90} = \frac{G}{100} = \frac{R}{2\pi}$

Answer

Let θ be the angle which is measure in degree, radian and grade

We know that 90°=1 right angle

$\Rightarrow 1^{\circ} = \frac{1}{90}$ right angle

$\Rightarrow D^{\circ} = \frac{D}{90}$ right angles

$\Rightarrow \theta = \frac{D}{90}$ right angle ......(1)

Also we know that ,π radians=2 right angles

$\Rightarrow 1^{c} = \frac{2}{\pi}$ right angle

$\Rightarrow R = \frac{2}{\pi} \times R$ right angles

$\Rightarrow \theta = \frac{2}{\pi} \times R$ right angles ......(2)

Also we know that, 100 grades=1 right angle

$\Rightarrow$1 grade$= \frac{1}{100}$ right angle

$\Rightarrow$G grade$= \frac{G}{100}$ right angles

$\Rightarrow \theta = \frac{G}{100}$ right angles ......(3)

From (1),(2) and (3)

$$\frac{D}{90} = \frac{2R}{\pi}$$

$$= \frac{G}{100}$$

2. Question

Mark the correct alternative in the following:

If the angles of a triangle are in A.P., then the measure of one of the angles in radians is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer

Here, angles of triangle are in A.P.

So, Let angles of triangle are a,a+d,a+2d.

We know that, sum of angles of triangle is π.

$\therefore$ a+a+d+a+2d=π

$\therefore$ 3a+3d=π

$\therefore$ 3(a + d) =π

$\therefore a + d = \frac{\pi}{3}$

Also, by our assumption, a + d is one angle of triangle.

So, required measure of one of the angles is $\frac{\pi}{3}$.

3. Question

Mark the correct alternative in the following:

The angle between the minute and hour hands of a clock at 8:30 is

A. 80°

B. 75°

C. 60°

D. 105°

Answer

We know, in clock 1 rotation gives 360°

i.e. 60 minutes=360° and 12 hours=360°

So,1 minute=6° and 1 hour=30°

Now, For hour hand:

8 hours=8×30°=240° and for another 30 minute (which is half of hour) =30°÷2=15°

i.e. angle traced by hour hand is 240°+15°=255°

Now, For minute hand:

30 minute=30×6°=180°

i.e. angle traced by minute hand is 180°.

So, the angle between hour hand and minute hand=255°-180°

=75°

4. Question

Mark the correct alternative in the following:

At 3:40, the hour and minute hands of a clock are inclined at

A. $\frac{2\pi^c}{3}$

B. $\frac{7\pi^c}{12}$

C. $\frac{13\pi^c}{18}$

D. $\frac{3\pi^c}{4}$

Answer

We know, in clock 1 rotation gives 360°

i.e. 60 minutes=360° and 12 hours=360°

So,1 minute=6° and 1 hour=30°

Now, For hour hand:

3 hours=3×30°=90° and for another 40 minute= (30°÷60)×40=20°

i.e. angle traced by hour hand is 90°+20°=110°

Now, for minute hand:

40 minute=40×6°=240°

i.e. angle traced by minute hand is 240°.

So, the angle between hour hand and minute hand=240°-110°

=130°

$= 130° \times \frac{\pi^c}{180}$

$= \frac{13\pi^c}{18}$

5. Question

Mark the correct alternative in the following:

If the arcs of the same length in two circles subtend angles 65° and 110° at the centre, then the ratio of the radii of the circles is

A. 22 : 13

B. 11 : 13

C. 22 : 15

D. 21 : 13

Answer

Let radius of two circles be the r_1 and r_2

Let θ_1 and θ_2 be the subtend angles of arcs of two circles

i.e. θ_1=65° and θ_2=110°

We know that arc length,

l = r × θ

Here, arc lengths of two circles are same.

$\therefore r_1 \times \theta_1 = r_2 \times \theta_2$

$\therefore \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{110}{65}$

$\therefore \frac{r_1}{r_2} = \frac{11 \times 2}{13}$

$\therefore r_1 : r_2 = 22:13$

6. Question

Mark the correct alternative in the following:

If OP makes 4 revolutions in one second, the angular velocity in radians per second is

A. π

B. 2 π

C. 4 π

D. 8 π

Answer

We know that, 1 revolution =2×π radians

Now, Angular velocity$= \frac{\text{Distance}}{\text{Time}}$

$= \frac{4 \text{ revolutions}}{1 \text{ second}}$

$= \frac{4 \times 2 \times \pi}{1}$

$=8\times\pi$

7. Question

Mark the correct alternative in the following:

A circular wire of radius 7 cm is cut and bent again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is

A. 50°

B. 210°

C. 100°

D. 60°

E. 195°

Answer

Here, radius of circular wire is r=7 cm

So, length of wire=2×π×r

=2×π×7

=14×π

Wire is cut and bent again into an arc of a circle of radius 12 cm.

So, length of arc=length of wire=14×π

We know, angle subtended by the arc is given by,

$$\theta = \frac{\text{length of arc}}{\text{radius}}$$

$$= \frac{14\pi}{12}$$

$$= \frac{14\pi}{12} \times \frac{180°}{\pi}$$

=210°

8. Question

Mark the correct alternative in the following:

The radius of the circle whose arc of length 15 π cm makes an angle of 3π/4 radian at the centre is

A. 10 cm

B. 20 cm

C. $11\frac{1}{4}$ cm

D. $22\frac{1}{2}$ cm

Answer

Here, arc length l=15π cm

Angle $\theta = \frac{3\pi}{4}$

We know, angle subtended by the arc is given by,

$$\theta = \frac{\text{length of arc}}{\text{radius}}$$

$\therefore$ radius $= \dfrac{l}{\theta}$

$= \dfrac{15\pi}{3\pi} \times 4$

$= 20$ cm